LAUNCHING TO ORBIT

What will you learn in this lab?

This lab will touch on the basics of rocket science. You will learn to calculate just how much effort it takes to send something into orbit by figuring out how much payload the NASA Space Shuttle can bring to a given orbit.

What do I need to bring to the Class with me to do this Lab?

For this lab you will need:
• A copy of this lab script
• A Scientific Calculator
• Graph Paper

Introduction

Getting to space is hard. In order be in orbit around the Earth, a satellite needs to go very fast (greater than 12,000 miles per hour), and very high (greater than 125 miles up). Ordinary airplanes can't go that high or fast, and even the fastest jet ever built, the SR-71, could only go 2,200 mph at 15 miles up. So, in order to put something in orbit, you need a rocket. But first, why so high and fast?

Orbital Velocity

The first person to really think about putting an object into orbit was Sir Isaac Newton. He imagined putting a cannon on top of a very high mountain and firing it horizontally. The cannonball would travel some distance, and then gravity would cause it to hit the ground. If Sir Isaac used a more powerful cannon, the ball will go further. The trick is, though, the Earth is round. So eventually, Newton's cannon would fire a ball fast enough that is would go right around the Earth and hit the back of the cannon that fired it. Newton called this speed **circular orbital velocity**, and wrote an equation for it:

$$v_{orb} = \sqrt{\frac{G\,M_{Earth}}{R_{Earth} + h}}$$

Where v_{orb} is orbital velocity, G is the gravitational constant, M_{Earth} and R_{Earth} are the mass and radius of the Earth, and h is height of the orbit.
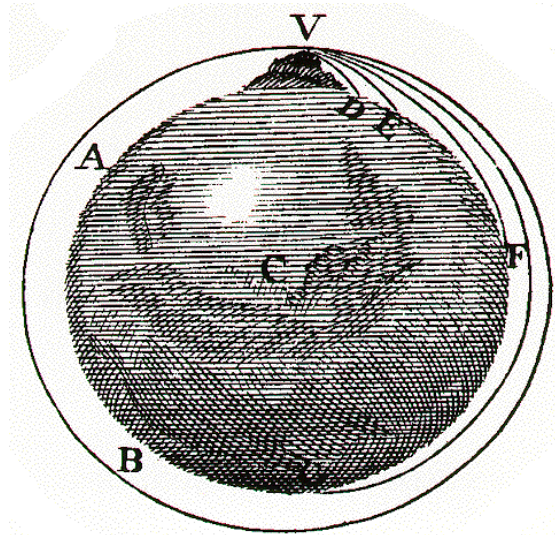
Newton's Orbital Cannon, from *Philosophiæ Naturalis Principia Mathematica*, 1687

The problem with Newton's Cannon is that if you fired it from any real mountain, the air drag would slow the cannonball down before it got around the Earth. So, to actually put an object in to orbit, you need to first bring it above most the Earth's atmosphere. In the impact cratering lab, we assumed that the velocity at which a dropped object hits the ground is $v_{alt} = \sqrt{2\,g\,h}$, where h is the height it falls and g is the gravitational acceleration. We can then estimate the amount of velocity it takes to bring an object to an orbital altitude with the same equation (this works for low orbits because gravity is almost the same as on the ground).

Finally, to find the actual velocity requirement for our space rocket, we need to take into account the fact that the rocket climbs at an angle into space, rather than straight vertical and the horizontal, as well as for the air drag the rocket will plow through on its way up. Putting this all together:

$$G * M_{Earth} = 4 * 10^5 \; km^3/s^2 \quad R_{Earth} = 6400\;km \quad g = 0.01\;km/s^2$$

$$v_{orb}{}^2 = \frac{G * M_{Earth}}{R_{Earth} + h} \qquad v_{alt}{}^2 = 2 * g * h$$

$$\Delta v = (1\;km/s) + \sqrt{v_{orb}{}^2 + v_{alt}{}^2}$$

The Rocket Equation

Now that we know how much total velocity it takes to get to orbit, we need to know how big a rocket it takes to get there. Rockets work on the principle of conservation of momentum. This concept (also Newton's idea), states that the total mass times velocity of an object (called its momentum) should always stay the same, even if the object breaks apart. Rockets make use of this principle by throwing material (called propellant) in one direction, and thus getting pushed in the opposite direction. The Russian scientist Konstantin Tsiolkovsky codified this idea with **the rocket equation**:

$$\Delta v = v_e \, Ln \left(\frac{M_{empty} + M_{propellant}}{M_{empty}} \right)$$

Where Δv is the velocity of the rocket after it has burnt all of its fuel, v_e is the exhaust velocity of the rocket, M_{empty} is the mass of the rocket without any fuel, and $M_{propellant}$ is the mass of fuel loaded into the rocket.

The NASA Space Shuttle
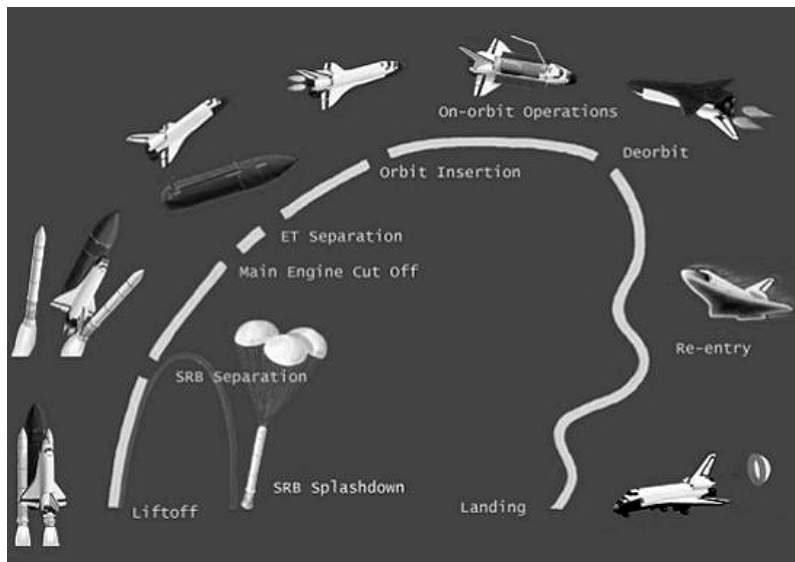
The largest and most complicated rocket flying today is the NASA Space Shuttle. The Shuttle essentially consists of four separate rockets bolted together and running at the same time to climb to orbit. The most recognizable part of the Shuttle is the Orbiter, which looks like a large white airplane and has a cavernous payload bay to carry cargo and seven astronauts to orbit and back. The orbiter, though, has no fuel tanks for its three main hydrogen-powered engines; instead, it draws from the even larger External Tank. Even still, the external tank does not have quite enough fuel to reach orbit, and so is complemented by two enormous Solid Rocket Boosters. These are less efficient than the orbiter's engines, but are the most powerful rockets

ever built. So, the SRBs allow the shuttle to plow through the atmosphere quickly, and then the more efficient hydrogen engines bring the orbiter and external tank to orbital velocity.

We can't use the Tsiolkovsky rocket equation by itself to model the shuttle, as the SRBs have a different exhaust velocity than the main engines, and come off when the external tank is still 3/4 full. But we can get around that by adding up two separate rocket equations:

$$\Delta v_1 = v_{e,1}\, Ln\left(\frac{payload + Orbiter_{empty} + Orbiter_{propellant} + SRB_{empty} + SRB_{propellant}}{payload + Orbiter_{empty} + 0.75\, Orbiter_{propellant} + SRB_{empty}}\right)$$

$$\Delta v_2 = v_{e,2}\, Ln\left(\frac{payload + Orbiter_{empty} + 0.75\, Orbiter_{propellant}}{payload + Orbiter_{empty}}\right)$$

Adding in real values for the Space Shuttle, these equations simplify down to:

$$\Delta v_1 = 2.81\, \frac{km}{s}\, Ln\left(\frac{payload + 2051\ tonnes}{payload + 863\ tonnes}\right)$$

$$\Delta v_2 = 4.44\, \frac{km}{s}\, Ln\left(\frac{payload + 681\ tonnes}{payload + 136\ tonnes}\right)$$

$$\Delta v = \Delta v_1 + \Delta v_2$$

Lockheed-Martin Atlas V:

The Space Shuttle is very large and very expensive to launch. So, smaller satellites, and larger ones that don't need help from astronauts, are typically launched on smaller expendable launch vehicles, like the Atlas V or Delta IV. NASA commonly uses the Atlas V model 401, which has the rocket equations:

$$\Delta v_1 = 3.32\, \frac{km}{s}\, Ln\left(\frac{payload + 334\ tonnes}{payload + 49\ tonnes}\right)$$

$$\Delta v_2 = 4.42\, \frac{km}{s}\, Ln\left(\frac{payload + 27\ tonnes}{payload + 6\ tonnes}\right)$$

$$\Delta v = \Delta v_1 + \Delta v_2$$

Part 1:

Using the equations given on page 2, fill in the table to find the Δv needed to reach each orbit:

Altitude	v_{orb}^2	v_{alt}^2	Orbital Δv
200 km			
300 km			
400 km			
500 km			
600 km			

Part 2:

Using the equations given on page 4, fill in the table to find the Δv that the **Space Shuttle** can provide for each payload:

Payload	$Δv_1$	$Δv_2$	Rocket Δv
25 tonnes			
20 tonnes			
15 tonnes			
10 tonnes			
5 tonnes			

Using the equations given on page 4, fill in the table to find the Δv that the **Atlas V** can provide for each payload:

Payload	Δv_1	Δv_2	Rocket Δv
12 tonnes			
10 tonnes			
8 tonnes			
5 tonnes			
2 tonnes			

Part 3:

Plot the Payload versus Rocket Δv for both the Space Shuttle and Atlas V. Which is rocket is optimized for low orbits? Which for higher orbits?

Additional Questions:

1. In 1990, the Hubble Space Telescope was delivered by the Space Shuttle *Discovery* to a 600 km orbit. This was the highest any Shuttle had ever flown, and used all the Δv *Discovery* had to offer. Based on your calculations above, approximately what is the mass of the Hubble Space Telescope?

2. The first private manned spacecraft, the Scaled Composites SpaceShipOne, had a total Δv of 1.7 km/s. Could it reach orbit? Why or why not?

3. Expendable launch vehicles like the Atlas V tend to have a much lighter empty mass than the Shuttle. Why is this useful?

4. Many communications satellites are in geosynchronous orbit, which means they always stay above the same point on the Earth's equator. The Δv needed to reach these orbits is about 10.7 km/s. Can the Shuttle fly to geosynchronous orbit? About how much payload can an Atlas V deliver there?

5. The speed necessary to leave Earth completely and go to either the Moon or Mars is called escape velocity, and the Δv needed to reach it is about 12 km/s. The NASA Lunar Reconnaissance Orbiter (which has an ASU-run camera) will be launched on an Atlas V to the Moon; about how much does it weigh?